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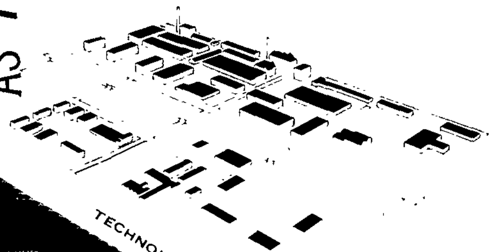

TECHNOLOGY CENTER

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TWO-CONDUCTOR LOW-PASS TRANSMISSION LINE THEORY

U. S. Naval Weapons Laboratory
Dahlgren, Virginia

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31 March 1963

Contract No. N178-7927
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ABSTRACT

A field analysis of a two-layer line was performed. It was shown that a TEM mode cannot propagate in such a structure. Instead, a TM mode will exist. However, at frequencies where the dimensions of the line are small compared to a wavelength, the TEM approximation may be used to determine the attenuation characteristics of such a line.

Commercially fabricated low-pass transmission lines were tested to determine how well the design specifications of these lines was met in a practical configuration. Two types of lines were fabricated. One used a conducting polyethelyne as the dielectric while the other utilized a conducting rubber compound for the same purpose. Due to contact problems between the two conductors and the dielectric material, neither of the lines met the design specifications. However, the shape of the attenuation versus frequency curve was as would be expected for a line with a shunt conductance lower than was the actual case. Improvement in the outer conductor contact enabled the response curve to be shifted down in frequency so that it more nearly approached the theoretical response. Several ferrite lines were also constructed and tested. These lines had an attenuation which varied in proportion to the square root of frequency.

TWO-CONDUCTOR LOW-PASS TRANSMISSION LINE THEORY

U. S. Naval Weapons Laboratory
Contract No. N178-7927

I. INTRODUCTION

This is the seventh quarterly report on ARF Project No. E167 entitled "Two-Conductor Low-Pass Transmission Line Theory." This program is being conducted for the U. S. Naval Weapons Laboratory under Contract No. N178-7927. The program covers a research study and a theoretical analysis of two-conductor transmission line theory.

It is desired to develop criteria for lines which will allow a desired audio control signal to be delivered to an electroexplosive device while preventing unwanted RF energy from being propagated down the line. Specifications call for a target goal of a line which has 3 db/meter of attenuation at 20 kc while attaining an attenuation of 60 db/meter as rapidly as possible as the frequency is increased.

The intial phases of the program were involved with theoretical analyses of lines which might be used to attain the desired goals. Later phases were concerned with the fabrication of simple laboratory models of several of these line types to determine if the theory used was satisfactory. Included in this latter portion of the work were investigations of materials which might be used in the fabrication of such lines.

Work during the seventh quarter has been carried out in two areas. The first of these involved a field analysis of the two-layer line in order to determine whether the circuit model previously proposed for this line was valid. Indications are that, at least for low frequencies, the approach previously used to determine the response of such a line is justified. Experimental determination of the response of commercially fabricated LG lines was the second main area of interest during the last quarter. Also investigated was the effect of using various techniques to supply an outer conductor to the line.

II. FIELDS IN A TWO-LAYER COAXIAL LINE

A. Introduction

The analysis of a transmission line is usually based on the assumption that the structure can maintain a TEM wave. This is convenient since under these conditions, distributed circuit elements can be associated with the line (R, L, G, C). For modes other than the TEM, it is necessary to use a field theory approach, solving Maxwell's equations under the constraints imposed by the boundary conditions.

The validity of describing the modal structure as having purely transverse electric and magnetic fields must be justified. It cannot be assumed that all two conductor structures will support such a mode. It can be shown that lines constructed of physical conductors and dielectrics cannot support a pure TEM mode, but must have field components in the direction of propagation. This is true for the single layer coaxial line when the conductors have losses as well as for the two-layer line even if the conductors have no losses.

The following analysis shows the spatial requirements of the fields for the true TEM case, a mode that can be supported in the two-layer line, and the relative magnitude of the axial field component in this two-layer line. The conditions under which the fields in the two-layer line can be approximated as being wholly transverse are also given, along with the attenuation and phase shift with this approximation.

B. The One Layer Line

The fields in the layer between the conductors of a coaxial line must obey Maxwell's Equations. If this region is charge free, isotropic, and homogeneous,

$$\nabla \times \mathbb{E} = -j\omega\mu \mathbb{H} \tag{1}$$

$$\nabla \times \mathbb{H} = j\omega\hat{\epsilon}\mathbb{E} \tag{2}$$

$$\nabla \cdot \mathbb{H} = 0 \tag{3}$$

$$\nabla \cdot \mathbb{E} = 0 \tag{4}$$

where $\hat{\epsilon}$ is the complex permitivity, given by

$$\hat{\epsilon} = \epsilon + \frac{\sigma}{j\omega} \tag{5}$$

The TEM solution to the above equations requires that

$$E_z = E_\varphi = H_r = H_z = 0 \tag{6}$$

With only one field component for each the electric and magnetic fields, the divergence equations reduce to

$$\frac{1}{r}\frac{\partial H_\varphi}{\partial \varphi} = 0 \tag{7}$$

and

$$\frac{1}{r}\frac{\partial (rE_r)}{\partial r} = 0 \tag{8}$$

Consequently,

$$H_\varphi = f_1(r, z) \tag{9}$$

and

$$E_r = \frac{1}{r} f_2(\varphi, z) \tag{10}$$

The functions f_1 and f_2 can be determined more explicitly through the curl equations. It is important to note, however, that if E_r is the only electric field present, than its r dependence must be of the form of $1/r$.

Four equations are generated from the curl equations. These are

$$\frac{\partial E_r}{\partial z} = -j\omega\mu H_\varphi \tag{11}$$

$$\frac{\partial E_r}{\partial \varphi} = 0 \tag{12}$$

$$-\frac{\partial H_\varphi}{\partial z} = j\omega\hat{\epsilon} E_r \tag{13}$$

$$\frac{\partial (r H_\varphi)}{\partial r} = 0 \tag{14}$$

Equation (12) shows that E_r has no ϕ dependence. Thus

$$E_r = \frac{f_2(\varphi, z)}{r} = \frac{f_3(z)}{r} \tag{15}$$

Equation (14) gives the r dependence of H_ϕ

$$r H_\varphi = r f_1(r, z) = f_4(z) \tag{16}$$

or

$$H_\varphi = \frac{f_4(z)}{r} \tag{17}$$

By differentiation of (11) and (13) with respect to z, the wave equations in $f_3(z)$ and $f_4(z)$ arise

$$\frac{d^2 f_3}{d z^2} = -\omega^2 \mu \hat{\epsilon} f_3 \tag{18}$$

$$\frac{d^2 f_4}{d z^2} = -\omega^2 \mu \hat{\epsilon} f_4 \tag{19}$$

which have as their solution,

$$f_3 = A e^{j\omega\sqrt{\mu\hat{\epsilon}}\, z} + B e^{-j\omega\sqrt{\mu\hat{\epsilon}}\, z} \tag{20}$$

and

$$f_4 = C e^{j\omega\sqrt{\mu\hat{\epsilon}}\, z} + D e^{-j\omega\sqrt{\mu\hat{\epsilon}}\, z} \tag{21}$$

Consequently, the only permissible form of a TEM solution in circular cylindrical coordinates is

$$E_r = \frac{1}{r}\left[A e^{j\omega\sqrt{\mu\hat{\epsilon}}\, z} + B e^{-j\omega\sqrt{\mu\hat{\epsilon}}\, z}\right] \tag{22}$$

$$H_\phi = \frac{1}{r}\left[C e^{j\omega\sqrt{\mu\hat{\epsilon}}\, z} + D e^{-j\omega\sqrt{\mu\hat{\epsilon}}\, z}\right] \tag{23}$$

C. The Two Layer Line

Consider a two layer line with the cross section shown in Figure 1. The boundary conditions on the magnetic field are that, on the interface between the two media,

$$H_{\phi_1} = H_{\phi_2} \tag{24}$$

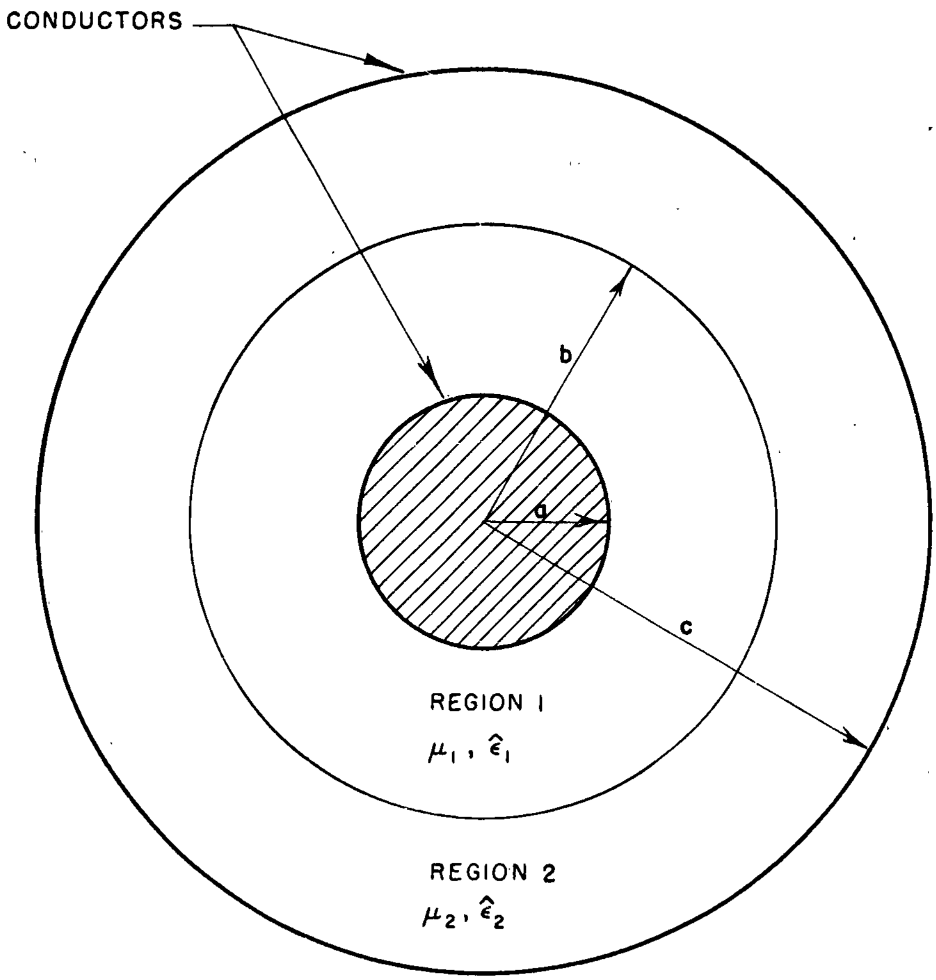

FIG. 1 TWO-LAYER COAXIAL TRANSMISSION LINE, CROSS-SECTIONAL VIEW

Due to the continuity of current at the interface,

$$\hat{\epsilon}_1 E_{r_1} = \hat{\epsilon}_2 E_{r_2} \tag{25}$$

In addition, the phase velocities must be the same in both regions. In region no. 1, let

$$f_4(z)\Big|_{\text{region 1}} = g_1(z) = C_1 e^{j\omega\sqrt{\mu\hat{\epsilon}}\, z} + D_1 e^{-j\omega\sqrt{\mu\hat{\epsilon}}\, z} \tag{26}$$

$$f_3(z)\Big|_{\text{region 1}} = h_1(z) = A_1 e^{j\omega\sqrt{\mu\hat{\epsilon}}\, z} + B_1 e^{-j\omega\sqrt{\mu\hat{\epsilon}}\, z} \tag{27}$$

and, in region no. 2, let

$$f_4(z)\Big|_{\text{region 2}} = g_2(z) = C_2 e^{j\omega\sqrt{\mu\hat{\epsilon}}\, z} + D_2 e^{-j\omega\sqrt{\mu\hat{\epsilon}}\, z} \tag{28}$$

and

$$f_3(z)\Big|_{\text{region 2}} = h_2(z) = A_2 e^{j\omega\sqrt{\mu\hat{\epsilon}}\, z} + D_2 e^{-j\omega\sqrt{\mu\hat{\epsilon}}\, z} \tag{29}$$

the boundary conditions impose the requirements that

$$g_1(z) = g_2(z) \tag{30}$$

$$\hat{\epsilon}_1 h_1(z) = \hat{\epsilon}_2 h_2(z) \tag{31}$$

Substituting (26) and (28) into (30) gives

$$C_1 e^{j\omega\sqrt{\mu_1\hat{\epsilon}_1}\, z} + D_1 e^{-j\omega\sqrt{\mu_1\hat{\epsilon}_1}\, z} = C_2 e^{j\omega\sqrt{\mu_2\hat{\epsilon}_2}\, z} + D_2 e^{-j\omega\sqrt{\mu_2\hat{\epsilon}_2}\, z} \tag{32}$$

This can only be true if, for all ω and z,

$$\mu_1 \hat{\epsilon}_1 = \mu_2 \hat{\epsilon}_2 \tag{33}$$

Equating real and imaginary parts, we get

$$\mu_1 \epsilon_1 = \mu_2 \epsilon_2 \tag{34}$$

and

$$\frac{\mu_1 \sigma_1}{\epsilon_1} = \frac{\mu_2 \sigma_2}{\epsilon_2} \tag{35}$$

Since the form of a TEM solution must be $\frac{f(z)}{r}$, the boundary conditions can only be matched at the interface of the two regions if (34) and (35) are satisfied. Even in the lossless case, $\sigma_1 = \sigma_2 = 0$, condition (34) will not, in general, be met. Consequently, a pure TEM mode, in general, will not exist in a two layer line.

D. Admissible Modes in the Two-Layer Line

Since a pure TEM mode cannot propagate in the two-layer line, there must be another mode with a cutoff frequency of zero which degenerates into the TEM solution when both layers are the same. A TM mode is shown to have these properties.

From (1) through (4), the vector wave equations can be obtained

$$\nabla^2 \mathbb{E} + \omega^2 \mu \hat{\epsilon} \mathbb{E} = 0 \tag{36}$$

$$\nabla^2 \mathbb{H} + \omega^2 \mu \hat{\epsilon} \mathbb{H} = 0 \tag{37}$$

Three scalar equations are generated from each vector equation of the form

$$\nabla^2 F_r - \frac{2}{r^2}\frac{\partial F_\phi}{\partial \phi} - \frac{F_r}{r^2} + \omega^2 \mu \hat{\epsilon} F_r = 0 \tag{38}$$

$$\nabla^2 F_\phi + \frac{2}{r^2}\frac{\partial F_r}{\partial \phi} - \frac{F_\phi}{r^2} + \omega^2 \mu \hat{\epsilon} F_\phi = 0 \tag{39}$$

and

$$\nabla^2 F_z + \omega^2 \mu \hat{\epsilon} F_z = 0 \tag{40}$$

where F represents either E or H. The Laplacian in circular cylindrical coordinates is given by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \tag{41}$$

The modal structure must be such that when both regions are the same, there is only a r component of E and a ϕ component of H. Both of these components must be independent of ϕ.

Expansion of the curl equations (1) and (2) show that H_ϕ and E_r are determined by E_z only, while E_ϕ and H_r are determined by H_z. Consequently, a solution containing only E_z, E_r, and H_ϕ is chosen with these three components independent of the ϕ coordinate. Since the equation for E_z is relatively simple in this case,

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2 \mu \hat{\epsilon} E_z = 0 \tag{42}$$

this field component is determined first. Since the z variation must be of the form $e^{-\gamma z}$, where γ is the propagation constant of the line, using separation of variables, one obtains

$$\frac{\partial^2 E_z'}{\partial r^2} + \frac{1}{r}\frac{\partial E_z'}{\partial r} + (\gamma^2 + \omega^2\mu\hat{\epsilon})E_z' = 0 \tag{43}$$

where the prime indicates that the Z variation has been removed from the dependent variable.

For brevity, let

$$\gamma^2 + \omega^2\mu\hat{\epsilon} = \beta_c^2 \tag{44}$$

Consequently

$$\frac{\partial^2 E_z'}{\partial r^2} + \frac{1}{r}\frac{\partial E_z'}{\partial r} + \beta_c^2 E_z' = 0 \tag{45}$$

which is just one form of Bessel's equation yielding

$$E_z = \left[A J_0(\beta_c r) + B Y_0(\beta_c r)\right] e^{-\gamma z} \tag{46}$$

where A and B are constants to be determined and J_o and Y_o are zero order Bessel Functions of the first and second kind respectively.

The transverse fields for the TM case are given by

$$\mathbb{E}_T = -\frac{\gamma}{\beta_c^2}\nabla_T E_z \tag{47}$$

$$\mathbb{H}_T = -j\frac{\omega\hat{\epsilon}}{\beta_c^2}\, \mathbf{i}_z \times \nabla_T E_z \tag{48}$$

where ∇_T is the transverse gradient. Evaluating (47) and (48)

$$E_r = -\frac{\gamma}{\beta_c}\left[A J_0'(\beta_c r) + B Y_0'(\beta_c r)\right] e^{-\gamma z} \tag{49}$$

$$H_\phi = -j\,\frac{\omega\hat{\epsilon}}{\beta_c}\left[A\,J_0'(\beta_c r) + B\,Y_0'(\beta_c r)\right]e^{-\gamma z} \tag{50}$$

where the primes over the Bessel functions denote derivatives with respect to the argument.

Equations (46), (49), and (50) apply to both regions. However, the constants assume different values in each region. Using the subscript n(= 1 or 2 depending on whether region 1 or 2 is being considered), we may write the fields in the line as

$$E_{z_n} = \left[A_n\,J_0(\beta_{c_n} r) + B_n\,Y_0(\beta_{c_n} r)\right]e^{-\gamma z} \tag{51}$$

$$E_{r_n} = -\frac{\gamma}{\beta_{c_n}}\left[A_n\,J_0'(\beta_{c_n} r) + B_n\,Y_0'(\beta_{c_n} r)\right]e^{-\gamma z} \tag{52}$$

$$H_{\phi_n} = -j\,\frac{\omega\hat{\epsilon}_n}{\beta_{c_n}}\left[A_n\,J_0'(\beta_{c_n} r) + B_n\,Y_0'(\beta_{c_n} r)\right]e^{-\gamma z} \tag{53}$$

The boundary conditions at r = a, b, c must be met. These conditions are

$$E_{z_1}(a, z) = 0 \tag{54}$$

$$E_{z_2}(c, z) = 0 \tag{55}$$

$$H_{\phi_1}(b, z) = H_{\phi_2}(b, z) \tag{56}$$

$$\hat{\epsilon}_1 E_{r_1}(b, z) = \hat{\epsilon}_2 E_{r_2}(b, z) \tag{57}$$

$$E_{z_1}(b, z) = E_{z_2}(b, z) \tag{58}$$

Examination of (52) and (53); shows that (56) and (57) are redundant.

Application of (54) or (55) expresses B_1 in terms of A_1

$$B_1 = -A_1 \frac{J_0(\beta_{c_1} a)}{Y_0(\beta_{c_1} a)} \tag{59}$$

Similarly, an expression for B_2 in terms of A_2 may be obtained. Equation (56) allows the expression of A_2 in terms of A_1

$$A_2 = A_1 \left[\frac{\hat{\epsilon}_1}{\hat{\epsilon}_2} \frac{\beta_{c_2}}{\beta_{c_1}} \frac{Y_0(\beta_{c_2} c)}{Y_0(\beta_{c_1} a)} \frac{J_0'(\beta_{c_1} b) Y_0(\beta_{c_1} a) - Y_0'(\beta_{c_1} b) J_0(\beta_{c_1} a)}{J_0'(\beta_{c_2} b) Y_0(\beta_{c_2} c) - Y_0'(\beta_{c_2} b) J_0(\beta_{c_2} c)} \right] \tag{60}$$

Equation (58) gives a conditional equation relating the two parameters β_{c_1} and β_{c_2}

$$\frac{J_0(\beta_{c_1} b) Y_0(\beta_{c_1} a) - J_0(\beta_{c_1} a) Y_0(\beta_{c_1} b)}{J_0'(\beta_{c_1} b) Y_0(\beta_{c_1} a) - J_0(\beta_{c_1} a) Y_0'(\beta_{c_1} b)} \frac{\beta_{c_1}}{\hat{\epsilon}_1} = \frac{J_0(\beta_{c_2} b) Y_0(\beta_{c_2} c) - J_0(\beta_{c_2} c) Y_0(\beta_{c_2} b)}{J_0'(\beta_{c_2} b) Y_0(\beta_{c_2} c) - J_0(\beta_{c_2} c) Y_0'(\beta_{c_2} b)} \frac{\beta_{c_2}}{\hat{\epsilon}_2} \tag{61}$$

The quantity of interst is, however, γ , the propagation factor. The propagation factor is related to β_{c_1} and β_{c_2} through (44). For either region, this becomes

$$\beta_{c_n}^2 = \gamma^2 + \omega^2 \mu_n \hat{\epsilon}_n \tag{62}$$

From (61) and (62), it is possible to determine γ in terms of the parameters of the line. The solution of (61) is a rather formidable job, but an approximation can be made that simplifies it considerably. If $\beta_{c_1}(b-a)$ and $\beta_{c_2}(c-b)$ are small,

$$|\beta_{c_1}(b-a)| = \left|\sqrt{\gamma^2 + \omega^2 \mu_1 \hat{\epsilon}_1}\right| (b-a) << 1 \tag{63}$$

this is equivalent, in the lossless case, to

$$\frac{b-a}{\lambda_1} << 1 \quad , \quad \frac{c-b}{\lambda_2} << 1 \tag{64}$$

When (64) holds, the Bessel functions evaluated at 'a' and 'c' can be approximated by the first two terms of a Taylor's series expanded around b. That is

$$J_0(\beta_{c_1} a) \approx J_0(\beta_{c_1} b) - (b-a)\beta_{c_1} J_0'(\beta_{c_1} b) \tag{65}$$

$$Y_0(\beta_{c_1} a) \approx Y_0(\beta_{c_1} b) - (b-a)\beta_{c_1} Y_0'(\beta_{c_1} b) \tag{66}$$

$$Y_0(\beta_{c_2} c) \approx Y_0(\beta_{c_2} b) + (c-b)\beta_{c_2} Y_0'(\beta_{c_2} b) \tag{67}$$

$$J_0(\beta_{c_2} c) \approx J_0(\beta_{c_2} b) + (c-b)\beta_{c_2} J_0'(\beta_{c_2} b) \tag{68}$$

Substituting these values into (61) and simplifying yields

$$\frac{\beta_{c_1}^2 (b-a)}{\hat{\epsilon}_1} + \frac{\beta_{c_2}^2 (c-b)}{\hat{\epsilon}_2} = 0 \tag{69}$$

Substituting (62) into (69) and solving for γ^2, we obtain

$$\gamma^2 = -\omega^2 \hat{\epsilon}_1 \hat{\epsilon}_2 \frac{(b-a)\mu_1 + (c-b)\mu_2}{(b-a)\hat{\epsilon}_2 + (c-b)\hat{\epsilon}_1} \tag{70}$$

Note that if ϵ_1 and ϵ_2 are real (the lossless case), γ is purely imaginary and propagation results. Note also that if $\hat{\epsilon}_1 = \hat{\epsilon}_2$ and $\mu_1 = \mu_2$ this is just the equation for the propagation factors for the TEM mode.

Using the approximations given by (65) through (68), it is of interest to determine the ratio of $\left|E_z/E_r\right|$ for a typical case. Consider E_z and E_r from (51) and (52).

$$\left|\frac{E_{z_n}}{E_{r_n}}\right| = \left|\frac{A_n J_0(\beta_{c_n} r) + B_n Y_0(\beta_{c_n} r)}{\frac{\gamma}{\beta_{c_n}}\left[A_n J_0'(\beta_{c_n} r) + B_n Y_0'(\beta_{c_n} r)\right]}\right| \tag{71}$$

Substitution of the value for B_1 into (71) yields

$$\left|\frac{E_{z_1}}{E_{r_1}}\right| = \left|\frac{\beta_{c_1}}{\gamma}\right| \left|\frac{J_0(\beta_{c_1} r) Y_0(\beta_{c_1} a) - Y_0(\beta_{c_1} r) J_0(\beta_{c_1} a)}{J_0'(\beta_{c_1} r) Y_0(\beta_{c_1} a) - Y_0'(\beta_{c_1} r) J_0(\beta_{c_1} a)}\right| \tag{72}$$

For small values of $\beta_{c_1} r$ and $\beta_{c_1} a$, the Bessel functions can be approximated by the first few terms of their expansions.

$$J_0(\beta_{c_1} r) \simeq 1 - \left(\frac{\beta_{c_1} r}{2}\right)^2 \tag{73}$$

$$J_0(\beta_{c_1} a) \simeq 1 - \left(\frac{\beta_{c_1} a}{2}\right)^2 \tag{74}$$

$$J_0'(\beta_{c_1} r) \simeq -\frac{\beta_{c_1} r}{2} + \frac{1}{2}\left(\frac{\beta_{c_1} r}{2}\right)^3 \tag{75}$$

$$Y_0(\beta_{c_1} r) \simeq \frac{2}{\pi}\left(k + \ln \frac{\beta_{c_1} r}{2}\right)\left[1 - \left(\frac{\beta_{c_1} r}{2}\right)^2\right] \tag{76}$$

$$Y_0(\beta_{c_1} a) \simeq \frac{2}{\pi}\left(k + \ln \frac{\beta_{c_1} a}{2}\right)\left[1 - \left(\frac{\beta_{c_1} a}{2}\right)^2\right] \tag{77}$$

$$Y_0'(\beta_{c_1} r) \simeq -\frac{2}{\pi}\left(k + \ln \frac{\beta_{c_1} r}{2}\right)\left[\frac{\beta_{c_1} r}{2} - \frac{1}{2}\left(\frac{\beta_{c_1} r}{2}\right)^3\right] + \frac{2}{\pi \beta_{c_1} r} \tag{78}$$

$$k = 0.5772157 = \text{Euler's constant} \tag{79}$$

Substituting these approximations into (72) and simplifying yields

$$\left|\frac{E_{z_1}}{E_{r_1}}\right| = \left|\frac{\beta_{c_1}}{\gamma}\right| \left|\frac{\left[1-\left(\frac{\beta_{c_1} r}{2}\right)^2\right]\frac{2}{\pi r}\ln\frac{a}{r}}{\frac{\beta_{c_1} r}{\pi}\left[1-\frac{1}{2}\left(\frac{\beta_{c_1} r}{2}\right)^2\right]\ln\frac{r}{a}+\frac{2}{\pi\beta_{c_1} r}}\right| \tag{80}$$

For $\beta_{c_1} r << 1$

$$\left|\frac{E_{z_1}}{E_{r_1}}\right| \cong \left|\frac{\beta_{c_1}}{\gamma}\right| \left|\beta_{c_1} r\right| \ln\frac{r}{a} \tag{81}$$

Let us consider the lossless case.

For

$$\mu_1 = \mu_2 = \mu_0 = 4\pi \times 10^{-7} \tag{82}$$

$$\epsilon_1 = \epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \tag{83}$$

$$\epsilon_2 = 9\epsilon_1 = \frac{1}{4\pi} \times 10^{-9} \tag{84}$$

$$\frac{b-a}{c-a} = 0.5 \tag{85}$$

$$a = 1 \text{ cm.} \tag{86}$$

$$b = 2a \tag{87}$$

(81) becomes

$$\left|\frac{E_{z_1}}{E_{r_1}}\right| = 2.76 \times 10^{-11}\,\omega \tag{88}$$

Thus it can be seen that the component of field in the direction of propagation can be neglected for frequencies less than five megacycles. The TEM approximation should be valid at lower frequencies.

Another expression for the propagation factor can be obtained using a different set of approximations for the Bessel Functions. For small values of argument, the Bessel Functions can be replaced by the first few terms of their power series expansions

$$J_0(x) \approx 1 - \left(\frac{x}{2}\right)^2 \qquad (89)$$

$$J_0'(x) \approx -\frac{x}{2} + \frac{1}{2}\left(\frac{x}{2}\right)^3 \qquad (90)$$

$$Y_0(x) \approx \frac{2}{\pi}\left(k + \ln\frac{x}{2}\right)\left[1 - \left(\frac{x}{2}\right)^2\right] \qquad (91)$$

$$Y_0'(x) \approx -\frac{2}{\pi}\left(k + \ln\frac{x}{2}\right)\left[\frac{x}{2} - \frac{1}{2}\left(\frac{x}{2}\right)^3\right] + \frac{2}{x} \qquad (92)$$

where

$$k = 0.5772157 \qquad \text{Euler's constant} \qquad (93)$$

Only the significant terms of the expansions will be used. For the case where

$$x = |\beta_{c_1} b| << 1 \qquad (94)$$

and

$$x = |\beta_{c_2} c| << 1 \qquad (95)$$

the significant terms in (89) through (93) are

$$J_0(x) \simeq 1 \tag{96}$$

$$J_0'(x) \simeq -\frac{x}{2} \tag{97}$$

$$Y_0(x) \simeq \frac{2}{\pi}\left(k + \ln\frac{x}{2}\right) \tag{98}$$

$$Y_0'(x) \simeq \frac{2}{x} \tag{99}$$

Substitution of (96) through (99) into (61) gives

$$\frac{\beta_{c_1}}{\epsilon_1} \frac{\ln\frac{\beta_{c_1} a}{2} - \ln\frac{\beta_{c_1} b}{2}}{\frac{\beta_{c_1} b}{2}\ln\frac{\beta_{c_1} a}{2} - \frac{1}{\beta_{c_1} b}} = \frac{\ln\frac{\beta_{c_2} c}{2} - \ln\frac{\beta_{c_2} b}{b}}{-\frac{\beta_{c_2} b}{2}\ln\frac{\beta_{c_2} c}{2} - \frac{1}{\beta_{c_2} b}} \frac{\beta_{c_2}}{\epsilon_2} \tag{100}$$

For small values of $\beta_{c_1} b, \beta_{c_1} a, \beta_{c_2} b, \beta_{c_2} c$

$$\left|\frac{\beta_{c_1} b}{2}\ln\frac{\beta_{c_1} a}{2}\right| << \frac{1}{\beta_{c_1} b} \tag{101}$$

and

$$\left|\frac{\beta_{c_2} b}{2}\ln\frac{\beta_{c_2} c}{2}\right| << \frac{1}{\beta_{c_2} b} \tag{102}$$

We may write (100) as

$$\frac{\beta_{c_1}^2}{\epsilon_1}\ln\frac{a}{b} = \frac{\beta_{c_2}^2}{\epsilon_2}\ln\frac{c}{b} \tag{103}$$

Using (62) and rearranging, we get

$$\gamma^2\left(\frac{1}{\hat{\epsilon}_1}\ln b/a + \frac{1}{\hat{\epsilon}_2}\ln c/b\right) = -\omega^2\left(\mu_2 \ln c/b + \mu_1 \ln b/a\right) \quad (104)$$

or

$$\gamma^2 = -\omega^2 \frac{\mu_2 \ln c/b + \mu_1 \ln b/a}{\frac{1}{\hat{\epsilon}_1}\ln b/a + \frac{1}{\hat{\epsilon}_2}\ln c/b} \quad (105)$$

This can be split into real and imaginary parts and α and β found. To split into real and imaginary parts, note that

$$\hat{\epsilon} = \epsilon - \frac{j\sigma}{\omega} \quad (5)$$

Consequently,

$$\gamma^2 = -\omega^2 \frac{\mu_2 \ln c/b + \mu_1 \ln b/a}{\frac{\ln b/a}{\epsilon_1 - j\frac{\sigma_1}{\omega_1}} + \frac{\ln c/b}{\epsilon_2 - j\frac{\sigma_2}{\omega}}} \quad (106)$$

Before proceeding, it will be shown that the expression given by (106) is just that for two series connected transmission lines. Consider a pair of concentric lines as shown in Figure 2.

The space between $r = b$ and $r = c$ is considered as one line, and that between $r = a$ and $r = b$ another. Connected in this way, these lines have the equivalent differential circuit shown in Figure 3. The static inductance, capacitance, and conductance for the coaxial lines are

$$L_1 = \frac{\mu_1}{2\pi} \ln b/a \quad (107)$$

$$L_2 = \mu_2/2\pi \ln c/b \quad (108)$$

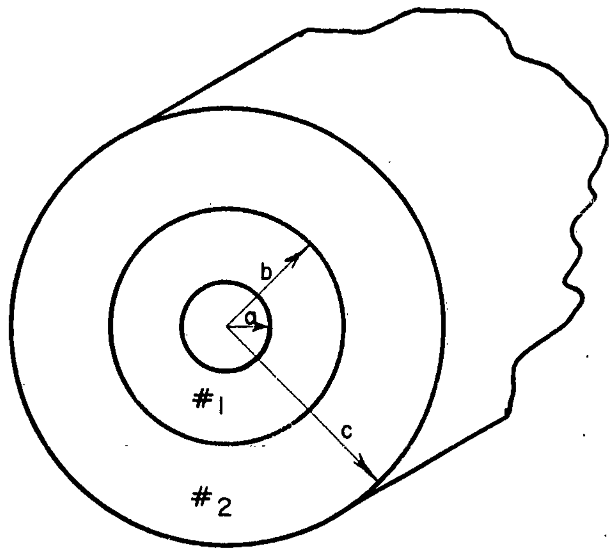

FIG. 2 CONCENTRIC TRANSMISSION LINES

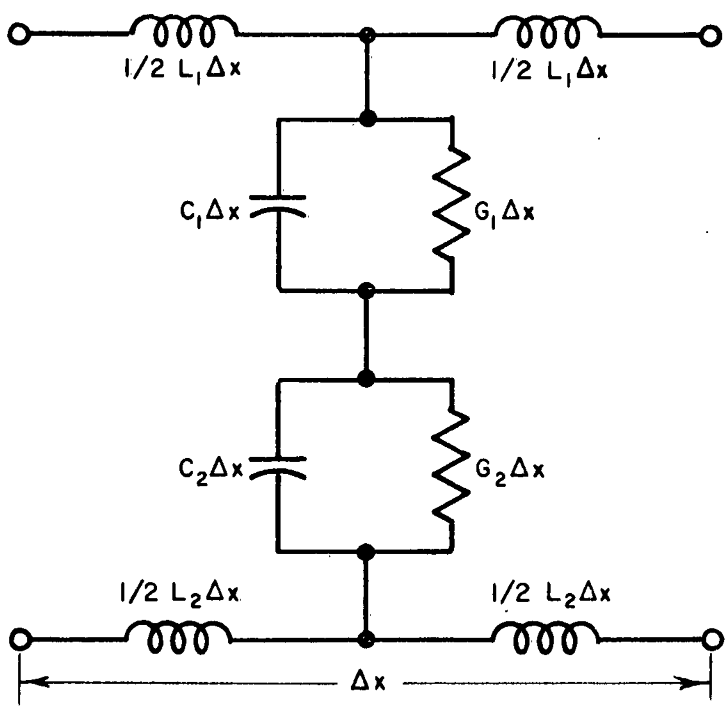

FIG. 3 DIFFERENTIAL CIRCUIT

$$C_1 = 2\pi\epsilon_1 \frac{1}{\ln b/a} \tag{109}$$

$$C_2 = 2\pi\epsilon_2 \frac{1}{\ln c/b} \tag{110}$$

$$G_1 = 2\pi\sigma_1 \frac{1}{\ln b/a} \tag{111}$$

$$G_2 = 2\pi\sigma_2 \frac{1}{\ln c/b} \tag{112}$$

The propagation factor is given by

$$\gamma^2 = j\omega L(G + j\omega C) = j\omega(L_1 + L_2)\frac{1}{\frac{1}{G_1 + j\omega C_1} + \frac{1}{G_2 + j\omega C_2}} \tag{113}$$

$$\gamma^2 = \frac{j\frac{\omega}{2\pi}\left[\mu_1 \ln b/a + \mu_2 \ln c/b\right]}{\frac{\ln b/a}{2\pi(\sigma_1 + j\omega\epsilon_1)} + \frac{\ln c/b}{2\pi(\sigma_2 + j\omega\epsilon_2)}} \tag{114}$$

or

$$\gamma^2 = -\omega^2 \frac{\mu_1 \ln b/a + \mu_2 \ln c/b}{\frac{\ln b/a}{\epsilon_1 - j\frac{\sigma_1}{\omega}} + \frac{\ln c/b}{\epsilon_2 - j\frac{\sigma_2}{\omega}}} \tag{115}$$

which is just (106). Therefore, the first term approximations of the Bessel Functions give the TEM case.

The frequency dependence of the attenuation for the TEM approximation is determined by finding the real part of the square root of (113).

Rearranging (113)

$$\gamma^2 = -\omega^2 \frac{L_1 + L_2}{\frac{1}{C_1 + \frac{G_1}{j\omega}} + \frac{1}{C_2 + \frac{G_2}{j\omega}}} \tag{116}$$

or

$$\gamma^2 = \frac{-\omega^2(L_1 + L_2)\left[C_1C_2 - \frac{G_1G_2}{\omega^2} - j\frac{1}{\omega}(G_1C_2 + C_1G_2)\right]\left[C_1 + C_2 + j\frac{1}{\omega}(G_1 + G_2)\right]}{(C_1 + C_2)^2 + \frac{(G_1 + G_2)^2}{\omega^2}} \tag{117}$$

Consequently

$$\gamma^2 = \frac{-\omega^2(L_1+L_2)\left\{\left(C_1C_2 - \frac{G_1G_2}{\omega^2}\right)(C_1+C_2) + \frac{1}{\omega^2}(G_1C_2 + C_1G_2)(G_1+G_2) + j\frac{1}{\omega}\left[\left(C_1C_2 - \frac{G_1G_2}{\omega^2}\right)(G_1+G_2) - (C_1+C_2)(G_1C_2 + C_1G_2)\right]\right\}}{(C_1+C_2)^2 + \frac{(G_1+G_2)^2}{\omega^2}} \tag{118}$$

Noting that if

$$\gamma^2 = A + jB \tag{119}$$

$$\alpha = \left[\frac{1}{2}\left(\sqrt{A^2+B^2} + A\right)\right]^{1/2} \tag{120}$$

we find the attenuation factor to be given by

$$\alpha = \omega\left\{\frac{L_1+L_2}{2\left[(C_1+C_2)^2 + \left(\frac{G_1+G_2}{\omega}\right)^2\right]}\cdot\left[\sqrt{\left[C_1C_2(C_1+C_2) + \frac{1}{\omega^2}(G_2^2C_1 + G_1^2C_2)\right]^2 + \frac{1}{\omega^2}\left[\frac{G_1^2G_2}{\omega^2} + \frac{G_1G_2^2}{\omega^2} + C_1^2G_2 + C_2^2G_1\right]^2} - C_1C_2(C_1+C_2) - \frac{1}{\omega^2}(G_2^2C_1 + G_1^2C_2)\right]\right\}^{1/2} \tag{121}$$

III. EXPERIMENTAL RESULTS

A. Introduction

It was decided, as has been previously reported, to contact several cable manufacturers to discuss with them the feasibility of fabricating lines which would incorporate conducting dielectric materials. Several such manufacturers were contacted and it was decided to produce pilot runs of two such cables. The first of these utilized a conducting polyethelyne material produced by the DuPont Company. This line was fabricated by the Coleman Cable and Wire Company of River Grove, Illinois. The second type of line utilized a conducting silicon rubber dielectric. No cable manufacturers could be found who were equipped or willing to produce small quantities of such a cable. However, Moxness Products, Inc., Racine, Wisconsin, agreed to extrude the material upon a bare copper wire. Half of the total run thus produced was then braided by the Coleman Cable Company. In addition, half of the conducting polyethelyne line was also braided. Upon receipt of the lines, attenuation measurements were performed using the techniques outlined in Quarterly Report No. 3.[1] The results of these measurements will now be reported.

B. Commercially Fabricated Lines

The first type of line to be discussed is one which uses as the dielectric a conducting polyethelyne manufactured by the DuPont Company. This material was purchased in bulk form and extruded onto a solid copper center conductor by the Coleman Cable and Wire Company. The center

[1] Tobin, H. G., "Two-Conductor Electrical Transmission Line Theory," Quarterly Report No. 3, Armour Research Foundation, Chicago, Illinois, 25 March 1962.

conductor was 16 gage or 0.051 inches in diameter. The O. D. of the dielectric was nominally 0.51 inches. Approximately 100 feet of the cable was manufactured. One-half of this length was then covered with a conventional copper braid to provide the outer conductor. The attenuation curve subsequently discussed refers to this latter cable as the braided poly line.

The second type of line which was fabricated consisted of a dielectric of a conducting silicon rubber compound. The dimensions of this line were similar to those of the poly line. The extrusion of the rubber upon a solid copper wire was performed by Moxness Products, Inc. No cable manufacturer was found who was either equipped or willing to perform this work. Since this company had no facility for manufacturing cables, it was not capable of braiding the extruded rubber. However, arrangements were made with the Coleman Company to braid approximately one-half of the 65 feet of rubber line which was received. The attenuation curves shown later refer to this braided cable as the braided rubber line.

Braiding was performed upon only a segment of the fabricated cable since it was not known if the braiding would provide sufficient contact of the outer conductor to the dielectric to allow the design specifications of the line to be met. First measurements upon the braided cables indicated that the expected attenuation was not being obtained. In an attempt to determine if the cause of this low value of attenuation was insufficient contact, a line was prepared with an adhesive conducting outer layer. This layer was prepared by using a mixture of an adhesive silicon compound and a conducting silver paste. This mixture was then painted onto the rubber.

One difficulty which arose was the cracking of this layer. The rubber is very flexible and care was required to attempt to prevent any bending of the line both while the mixture was drying and while the measurements were being performed. The line prepared in this manner will be subsequently referred to as the silvered rubber line.

The attenuation of these lines was determined by measuring the open circuit input and output voltage of the line and using the ratio of these voltages as an indication of the attenuation. The results of these measurements upon each of the lines discussed are shown in Figure 4. Also shown for reference is a line which varies as the square root of frequency. It can be seen that each of the lines varied at approximately this rate, at least at frequencies above one megacycle.

The two braided lines used were each two meters in length, while the silvered rubber line used for the determination of attenuation was only one meter in length. This smaller length was used in order to minimize the fabrication problems discussed earlier. Additional lines were tested and the results correlated fairly well with those shown in Figure 4.

It can be seen that the response of the silvered rubber line is essentially of the same shape as that of the braided rubber line except for being shifted in frequency towards the low frequencies. This would seem to indicate that the contact obtained with the silvered line was better than was provided by the braid. Essentially, the effect of better contact is to approach the design values of the line. Although the lines were designed with a resistivity of approximately 70 ohm-centimeters, the effective value of conductance will not reflect this value of resistivity unless complete contact between the inner and outer conductors is obtained. A net value of

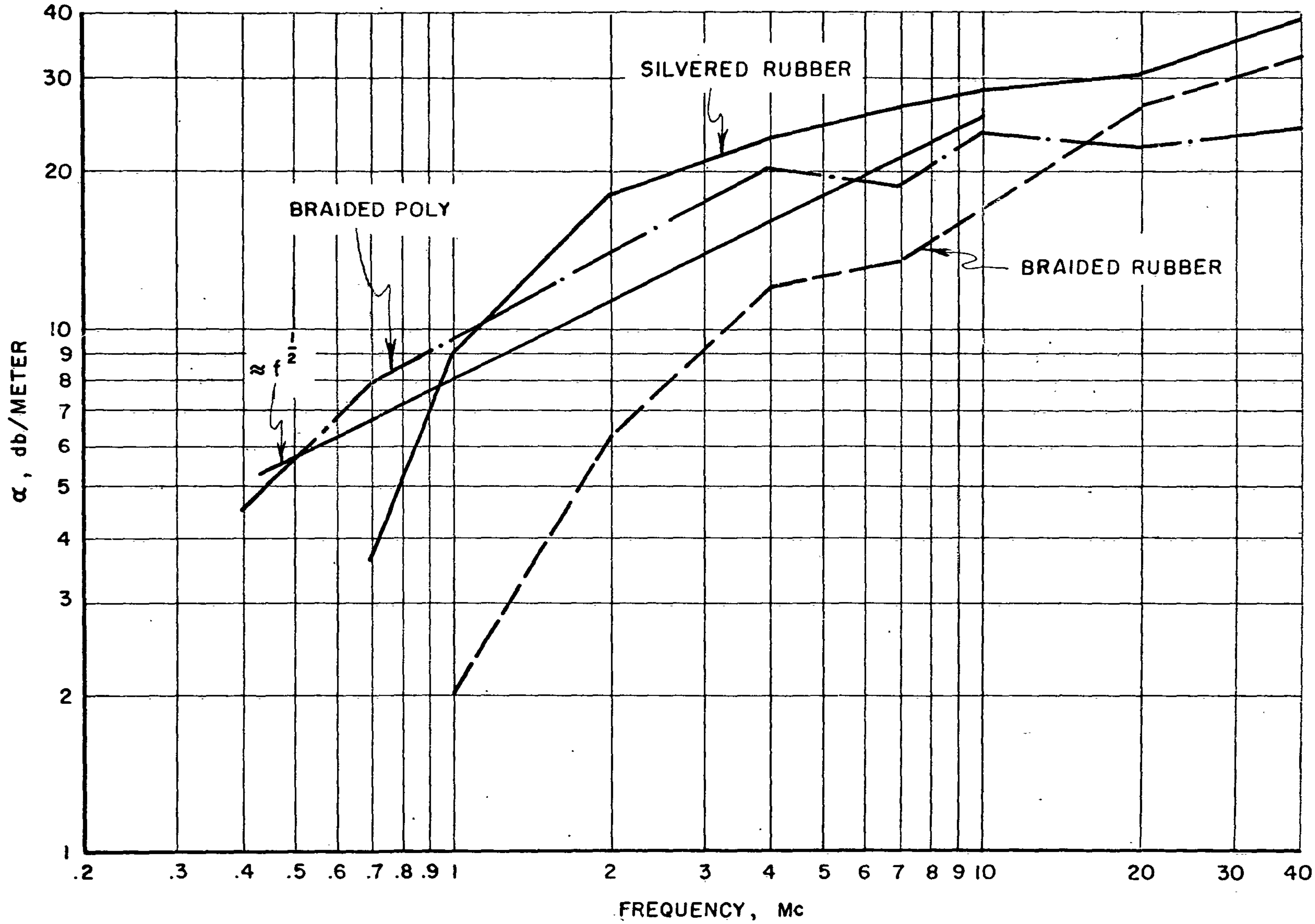

FIG. 4 RESPONSE OF FABRICATED LINES

conductance will be obtained which will be less than the desired value. From the attenuation curves, an estimate can be made of the effective resistivity obtained. From this effective resistivity, one may estimate how near one hundred percent contact is being approached.

The silvered rubber line gives an effective resistivity of about 300 ohm-cm., while the value obtained for the braided rubber line is 1850 ohm-cm.. The value obtained for the braided poly line is about the same as that of the silvered rubber line. From these figures, it can be seen that full benefit from the lines is still not being achieved. Further investigation of possible contact-enhancing fabrication techniques will be made in order to attempt to allow the design parameters of the line to be obtained. The data on the rubber line indicates an improvement in the effective resistivity of the line by a factor of five over that which has just the braid. An additional improvement of about the same ratio would be required to utilize the dissipation qualities of the line to their fullest. It is possible that this improvement will not be obtained by variations in the outer conductor only. Techniques for enhancing the contact between the inner conductor and the rubber will probably be required.

An alternative scheme might be used to obtain the desired attenuation characteristics. This would involve using a filler which would have a resistivity well below that which a perfect line would require to give the desired response. An attempt could then be made to control the contact so as to give an effective resistivity of 70 ohm-cm. Such a technique, while possible, would not be in the best engineering practice, however. Difficulty in obtaining any repeatability in the cable from one run to another or, even in the same wire, in maintaining constant percentage contact under bending conditions would lead to difficulties in predicting what the response of such a line would be.

The above values of effective resistivity of the various lines were computed under the assumption that the lines as fabricated represented ideal LG lines. That this is not case is indicated by the fact that, at low frequencies, the open circuit output voltage is greater than the input voltage. It can be shown, that, for an ideal LG line, such a situation is impossible. Since the lines are not ideal, it is of interest to compare their effective resistivities at dc. This value may be obtained from a measurement of the dc resistance between the inner and outer conductors. The values of resistivity computed by this measurement are 3400 ohm-cm for the braided poly line and 16,500 ohm-cm for the two rubber lines. Measurements were also made upon bulk specimens of the materials used. The values of resistivity obtained under this condition are 70 ohm-cm for the rubber and 10 ohm-cm for the poly. This latter value indicates that the line fabricated had an even lower value of resistivity than was specified and yet still did not approach the design values of attenuation.

C. Miscellaneous Lines

During the past quarter, two different types of ferrite lines were investigated. These lines differed only in the fabrication techniques used. The first type of line consisted of ferrite beads normally used to provide rf attenuation on power and filament leads. The dimensions of these cylindrical beads are a length of 0.296", an O. D. of 0.295", and an I. D. of 0.093." Initial tests of this type of line were made on specimens which were prepared by applying silver, air-drying paint to the inner and outer surfaces of the bead. The beads were then strung upon a center conductor. In some cases, the line was wrapped in a silvered, mylar tape to provide the outer conductor. A ten-inch section of line was prepared in this manner. Attenuation was initially noticed at approximately 450 kc. That is, the open circuit output voltage became less than the input voltage at that frequency. An attenuation of 0.3 db/cm was achieved at one megacycle. The attenuation rose to a value of approximately 1.5 db/cm at 10 mc. At higher frequencies the attenuation remained fairly constant at about this latter value.

A silver paint was obtained from the Hanovia Corporation which is used to provide conductive surfaces on ceramic materials. Additional beads were painted and a line fabricated as described above. The dc resistance of this line was 3,000 ohms. The attenuation of the line was 0.19 db/cm at 100 kc, 1 db/cm at 1 mc, and 4.66 db/cm at 40 mc. The beads were then fired at the recommended firing temperature of 1500° F. The dc resistance dropped to 450 ohms after the line was fired. A copper wire was soldered to the outer surface of the beads to provide continuity of the outer conductor. This version of the line exhibited less attenuation than the line mentioned above. No attenuation was observable until frequencies above 400 kc were reached. A peak value of 1.45 db/cm was obtained at 10 mc.

The second type of ferrite line constructed was an extension of the conventional type just discussed. However, the surfaces of the bead perpendicular to the axis of the line were painted with a resistive paint. It was felt that such a technique might provide additional attenuation in two ways.

The first of these would be the additional shunt loading of the line. The second of these would be through reflection at the interfaces. It is not known what the relative magnitudes of these two effects would be.

After the beads were painted and fired with the resistive paint, the shunt dc resistance of the line decreased to 20 ohms. Attenuation effects were evident at as low as 40 kc. The attenuation at this frequency was 0.2 db/cm. The attenuation continued to rise as frequency increased and reached a value of 4.5 db/cm at 40 mc.

It is of interest to note that both the unfired but silvered line and the line with the resistive paint had approximately the same attenuation. Both of these lines also increased in attenuation at a rate which was about the square root of frequency. Further experiments are planned with these elements to determine if they might be used in a practical line design. If time permits, an attempt will be made to fabricate a ferrite-loaded line. Ferrite beads will be interconnected by flexible braid inner and outer conductors. The resistivity of the bead could be controlled or a resistive layer printed on the surface of the bead perpendicular to the surface. Between the beads, a spongy dielectric, preferably conducting, would be used to give both body and flexibility to the line.

IV. CONCLUSIONS

It has been shown that, in general, a pure TEM mode cannot exist in a two-layer line. The equations for the field components of a TM mode that can exist has been developed. The conditional equation for the propagation factor, γ , for this mode was derived. It has been shown that with the restriction that the dimensions be small compared to a wavelength, the first order approximation to the TM field structure is the TEM case. The equations for the attenuation and phase shift with this approximation have been derived. The ratio of the magnitude of the axial electric field to the radial electric field has been shown to be insignificant for a typical case.

Two versions of the LG line have been commercially fabricated. Measurement of the attenuation of these lines indicates that contact problems between the two conductors and the dielectric still exist. The feasibility of the use of such lines has been demonstrated, however. The attenuation of these lines varies as would be theoretically expected. The response of the lines is, however, shifted up in frequency. It is felt that once the problem of insuring contact is overcome, the desired attenuation should be achieved.

The use of ferrite beads in a transmission line should enable additional attenuation to be achieved. Model lines consisting of the beads alone allowed the attainment of 1 db of attenuation for each cm of bead length. Although the beads alone would not always be desirable, it should be possible to design a line which would incorporate periodic loading with the beads to give attenuation rates which vary more rapidly than is now achievable with the LG line alone . The technique of printing a conducting layer on the bead surfaces perpendicular to the cable axis is desirable from two viewpoints. First, it permits greater attenuation to be realized. Second, printing resistive surfaces to $\pm 20\%$ tolerances is a well established manufacturing process and may eliminate the need to control the resistivity of the ferrite.

V. FUTURE WORK

Several areas will be investigated during the last quarter of the program. The work of most importance will be concerned with the fabrication of LG lines. It is planned to fabricate additional samples of the conducting polyethelyne lines. These lines will have two important changes over those previously constructed. The overall size of the line will be decreased to give a line which will be more similar to one which would be of practical use. Present plans call for the fabrication of lines of the size of RG-59/U cable. In addition to the decrease in the cable size, one other change will be made in the cable. Rather than a solid center conductor, a stranded wire will be used. This wire, while having the same average diameter as the solid wire, should offer a greater surface area to the dielectric. This will enable contact to be made at a greater percentage of the inner surface of the dielectric and should allow the design values of the line parameters to be closely approached.

Simultaneous study of the contact problem will be made at Armour Research Foundation. Initially, it is planned to immerse a cable in a highly-conducting salt solution. This solution will be used to act as the outer conductor of the cable. The use of a wetting agent in the salt solution should enable contact to be made to the dielectric over the entire outer surface. Attenuation tests upon this cable will enable a determination of the practicality of the braid as an outer conductor. If these tests indicate difficulty in attaining contact on the inner conductor, it is planned to prepare a line with an inner and outer conductor of mercury. This line will be for test purposes only.

In addition to the work on the LG cable, several experimental type lines will be studied. One of these will be a line which utilizes a ferrite-filled polymer as the dielectric. Several additional lines will also be studied. Included will be lines which incorporate as the dielectric some of the materials discussed in Quarterly Report No. 4.[1] As reported, several of these fillers appeared to attenuate at a rate more rapidly than the square root of frequency. The main difficulty with the materials used in those lines was the viscosity of the filler. The gelatinous salts used were semi-liquid at room temperature. In an attempt to devise a technique for utilizing the properties of these materials in a practical line, various solidifying agents will be used in conjunction with the gelatinous materials. It is hoped that such techniques will enable a practical line design to be obtained.

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[1] Tobin, H. G. "Two-Conductor Low-Pass Transmission Line Theory," Quarterly Report No. 4, Armour Research Foundation, Chicago, Illinois, pp. 6 - 9.

VI. IDENTIFICATION OF KEY PERSONNEL

The following is a listing of key personnel who have contributed to this program during the seventh quarter together with the approximate number of hours each has spent on the program during this quarter.

Name	Hours
J. E. Bridges, Manager	44
K. Gutfreund, Research Chemist	48
H. G. Tobin, Assistant Engineer	214
E. W. Weber, Associate Engineer	36

VII. LOGBOOK REFERENCES

Detailed laboratory data is contained in ARF Logbooks C11735, C12237, A12850, and C13380.

Respectfully submitted,

ARMOUR RESEARCH FOUNDATION
of Illinois Institute of Technology

H. G. Tobin

H. G. Tobin, Assistant Engineer

APPROVED:

J. E. Bridges, Manager
Electronic Compatibility

J. E. McManus, Assistant Director
Electronics Research